Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

This makes $r<2$ so that for systems (1)

$$-\frac{D}{4}=1,\ 2,\ 3,$$

and for systems (1′),

$$\frac{1-D}{4}=1,\ 2,\ 3,$$

are the possible cases. Investigating these cases using (14), we find that (4) obtains for the equations

$$w^2+1=0;\ w^2+2=0;\ w^2+w+1=0;\ w^2+w+2=0;\ w^2+w+3=0\ldots(15).$$

In these cases we have a greatest common divisor process.

If we define a prime number N of a system (1) or (1′) as one which has no factor n such that $1<|n|<|N|$, we have the theorem:

Every number N of one of the systems (15) can be decomposed essentially in but one way into prime factors. The non-essential variations are obtained frem the factors f such that $|f|=1$, which are the complete units.

The work here given completes the consideration of the case for a negative discriminant D, and agrees with the results of Dedekind, *Zahlentheorie*, fourth edition, page 450.

ON THE CHORD OF CONTACT OF TANGENTS TO A CONIC.

By W. D. LAMBERT, Washington, D. C.

If we wish to find the equation of the chord of contact of tangents drawn from a point $P_1\equiv(x_1, y_1)$, to the conic*

$$Ax^2+Bxy+Cy^2+Dx+Ey+F=0\ldots(1),$$

the straightforward way is to denote the unknown points of contact $P_2\equiv(x_2, y_2)$ and $P_3\equiv(x_3, y_3)$, and proceed to find the values of those coördinates. For this purpose we get two equations by substituting (x_2, y_2) and (x_3, y_3) in (1), and two more by expressing the fact that P_2 lies on the tangents through P_2 and P_3. These are

$$Ax_1x_2+\frac{B}{2}(x_1y_2+x_2y_1)+Cy_1y_2+\frac{D}{2}(x_1+x_2)+\frac{E}{2}(y_1+y_2)+F=0\ldots(2),$$

---

*I carry the work through for the general case, but as beginners are likely to be appalled by the mere length of an equation that is simple enough in principle, it is advisable in teaching to take at first a simpler special form of the equation of the conic.

$$Ax_1x_3+\frac{B}{2}(x_1y_3+x_3y_1)+Cy_1y_3+\frac{D}{2}(x_1+x_3)+\frac{E}{2}(y_1+y_3)+F=0\ldots(3).$$

These four equations give x_2, y_2, x_3, y_3 in terms of A, B, C, etc., and x_1, y_1. The required chord of contact is found by substituting their values in the formula for the straight line

$$\frac{y-y_2}{x-x_2}=\frac{y_3-y_2}{x_3-x_2}\ldots(4).$$

This direct solution is obvious in principle, but very tedious in the analytic work, and does not appear in text books. The favorite method in elementary treatises is to *assume* the result, namely, that the equation of the chord of contact of tangents to the conic (1) is

$$Ax_1x+\frac{B}{2}(xy_1+x_1y)+Cy_1y+\frac{D}{2}(x+x_1)+\frac{E}{2}(y+y_1)+F=0\ldots(5),$$

and then prove that (5) is a straight line passing through the points of contact P_2 and P_3. This process is certainly contrary to the spirit of *analytic* geometry, and seems obscure to many beginners.

The direct attack may be made easier by noting that we require not x_2, y_2, x_3, and y_3 themselves, but only certain combinations of them that are easy to find. Subtract (2) from (3) and factor so as to bring out x_3-x_2 and y_3-y_2. We may write the resulting equation in the form

$$\frac{y_3-y_2}{x_3-x_2}=-\frac{Ax_1+\frac{B}{2}y_1+\frac{D}{2}}{\frac{B}{2}x_1+Cy_1+\frac{E}{2}}.$$

Substitute this value in (4) and clear of fractions; after transposing the result may be written

$$Ax_1x+\frac{B}{2}(x_1y+xy_1)+Cy_1y+\frac{D}{2}x+\frac{E}{2}y-[Ax_1x_2+\frac{B}{2}(x_1y_2+x_2y_1)+Cy_1y_2]+$$
$$\frac{D}{2}x_2+\frac{E}{2}y_2=0\ldots(6).$$

Adding the identity (2) to (6), we get the required equation (5).